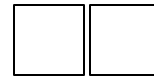


Prova de Conhecimentos Específicos

1ª QUESTÃO: (1,5 ponto)



Considere a matriz $A = \begin{pmatrix} \cos \theta & -\operatorname{sen} \theta \\ \operatorname{sen} \theta & \cos \theta \end{pmatrix}$. Determine:

- a) a transposta de A.
- b) a inversa de A.
- c) os autovalores de A, para $\theta = \frac{\pi}{4}$

Cálculos e respostas:

$$A = \begin{pmatrix} \cos \theta & -\operatorname{sen} \theta \\ \operatorname{sen} \theta & \cos \theta \end{pmatrix}$$

a)

$$A^T = \begin{pmatrix} \cos \theta & \operatorname{sen} \theta \\ -\operatorname{sen} \theta & \cos \theta \end{pmatrix};$$

$$b) A^{-1} = \frac{1}{\det A} (\operatorname{cof} A)^T$$

$$\det A = \cos^2 \theta + \operatorname{sen}^2 \theta = 1$$

$$\operatorname{cof} A = \begin{pmatrix} \cos \theta & -\operatorname{sen} \theta \\ \operatorname{sen} \theta & \cos \theta \end{pmatrix} \quad \operatorname{cof} A^T = \begin{pmatrix} \cos \theta & \operatorname{sen} \theta \\ -\operatorname{sen} \theta & \cos \theta \end{pmatrix}$$

$$A^{-1} = \quad = A^T$$

$$c) \theta = \pi/4 \Rightarrow A = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\det (A - \lambda I) = \det \begin{pmatrix} \frac{\sqrt{2}^2}{2} - \lambda & -1 \\ 1 & \frac{\sqrt{2}}{2} - \lambda \end{pmatrix} = \left(\frac{\sqrt{2}}{2} - \lambda \right)^2 + 1 = \frac{1}{2} - \sqrt{2}\lambda + \lambda^2 + 1$$

$$\lambda^2 - \sqrt{2}\lambda + \frac{3}{2} = 0 \quad = \frac{\sqrt{2} \pm \sqrt{2-6}}{2} = \frac{\sqrt{2} \pm 2i}{2} = \frac{\sqrt{2}}{2} \pm i$$

2ª QUESTÃO: (1,5 ponto)



Calcule $\int \frac{2x-3}{(x-1)(x-2)} dx$

Cálculos e respostas:

$$I = \int \frac{2x-3}{(x-1)(x-2)} dx$$

$$\frac{2x-3}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{Ax-2A+Bx-B}{(x-1)(x-2)} = \frac{(A+B)x + (-2A-B)}{(x-1)(x-2)}$$

$$\begin{cases} A+B=2 & A+B=2 \\ 2A-B=-3 & 2A+B=3 \end{cases}$$

$$A = 1$$

$$B = 1$$

$$I = \int \frac{dx}{x-1} + \int \frac{dx}{x-2} = \ln(x-1) + \ln(x-2) + C$$

$$= \ln \frac{x-1}{x-2} + C$$

3ª QUESTÃO: (1,0 ponto)



Dada a função f definida por $f(x, y, z) = \ln(x^2 + y^2 + \frac{xy}{z})$, determine seu vetor gradiente no ponto $(1, 2, -2)$.

Cálculos e respostas:

$$\nabla f = (f_x, f_y, f_z)$$

$$f_x = \frac{2x + y/z}{x^2 + y^2 + xy/z} \quad ; \quad f_y = \frac{2y + x/z}{x^2 + y^2 + xy/z}$$

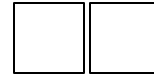
$$f_z = \frac{-xy/z^2}{x^2 + y^2 + xy/z}$$

$$f_x(1, 2, -2) = \frac{2 - 1}{1 + 4 - 1} = \frac{1}{4}$$

$$f_y(1, 2, -2) = \frac{4 - 1/2}{4} = \frac{7}{8}$$

$$f_z(1, 2, -2) = \frac{-1/2}{4} = -\frac{1}{8}$$

$$\nabla f(1, 2, -2) = \left(\frac{1}{4}, \frac{7}{8}, -\frac{1}{8} \right)$$



4ª QUESTÃO: (1,0 ponto)

Calcule $\int_R (x+y)dA$, sendo R a região do plano definida por
 $y \leq x, y \leq 2-x^2, y \geq 0$.

Cálculos e respostas:

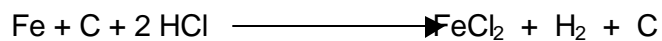
$$\begin{aligned}
 I &= \int_R (x+y)dA = & I &= \int_0^1 \int_y^{\sqrt{2-y}} (x+y) dx dy \\
 &= \int_0^1 \left(\frac{x^2}{2} + xy \right) \Big|_y^{\sqrt{2-y}} dy \\
 &= \int_0^1 \left(\frac{2-y}{2} + y\sqrt{2-y} - \frac{y^2}{2} + y^2 \right) dy \\
 &= \int_0^1 \left[y\sqrt{2-y} + 1 - \frac{y}{2} + \frac{y^2}{2} \right] dy \\
 \int_0^1 y\sqrt{2-y} dy &= -2 \int_{\sqrt{2}}^1 (2-u^2) du = -2 \int_{\sqrt{2}}^1 (2u^2 - u^4) du \\
 \left. \begin{array}{l} 2-y = u^2 \rightarrow y=0 \rightarrow u=\sqrt{2} \\ y=2-u^2 \quad y=1 \rightarrow u=1 \\ dy = -2udu \end{array} \right\} & & & = -2 \left[\frac{2u^3}{3} - \frac{u^4}{4} \right]_{\sqrt{2}}^1 \\
 & & & = -2 \left[\frac{2}{3} - \frac{1}{4} - \frac{2 \cdot 2 \sqrt{2}}{3} + 1 \right] \\
 & & & = -2 \left[\frac{17}{12} - \frac{4\sqrt{2}}{3} \right] \\
 & = -2 \left[\frac{17}{12} - \frac{4\sqrt{2}}{3} \right] + 1 - \frac{y^2}{2} \Big|_0^1 + \frac{y^3}{6} \Big|_0^1 & = & \\
 & = -2 \left[\frac{17}{12} - \frac{4\sqrt{2}}{3} + 1 - \frac{1}{2} + \frac{1}{6} \right]
 \end{aligned}$$



5ª QUESTÃO: (1,5 ponto)

Tratando-se 0,5 kg de ferro comercial (ferro-gusa) pelo HCl concentrado, obteve-se 200 litros de hidrogênio nas CNTP. Qual a percentagem de ferro puro na amostra de ferro-gusa?

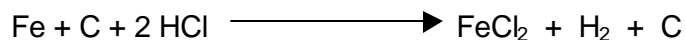
Dados:



Pesos Atômicos:

Fe → 56
 C → 12
 H → 1
 Cl → 35,5

Cálculos e respostas:



$$\begin{array}{r} 56 \text{ ————— } 22,4 \\ x \text{ ————— } 200 \end{array}$$

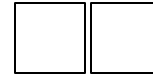
$$x = 500 \text{ g}$$

Cálculo do grau de pureza

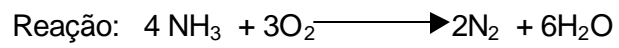
$$\begin{array}{r} \text{—————} \\ 500 \text{ ————— } 500 \\ 100 \text{ ————— } y \end{array}$$

$$y = 100 \% \text{ de ferro puro.}$$

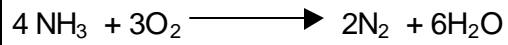
6ª QUESTÃO: (1,5 ponto)



Que volume de nitrogênio resulta, nas CNTP, da combustão de 50 litros de gás amoníaco?



Cálculos e respostas:



$$4 \times 22,4 \quad \text{-----} \quad 2 \times 22,4$$

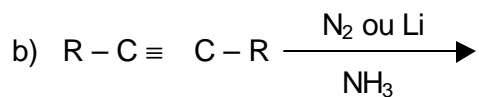
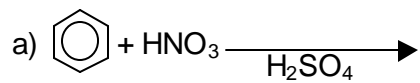
$$50 \quad \text{-----} \quad x$$

$$x = 25 \text{ litros}$$

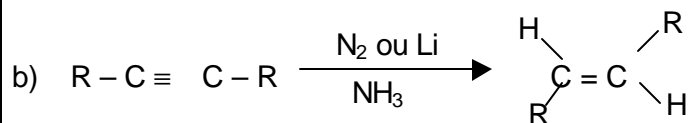
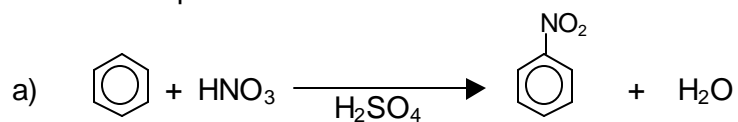


7ª QUESTÃO: (1,0 ponto)

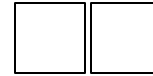
Complete as seguintes reações químicas:



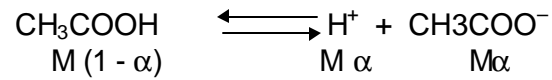
Cálculos e respostas:



8ª QUESTÃO: (1,0 ponto)



Calcule o grau de ionização de uma solução 0,2 M de ácido acético, sabendo que a constante de ionização do ácido a 25 °C é $1,8 \times 10^{-5}$ e



M → molaridade da solução

α → grau de ionização

Cálculos e respostas:

$$K = \frac{M\alpha^2}{1 - \alpha}$$

$$1,8 \times 10^{-5} = \frac{0,2\alpha^2}{1 - \alpha}$$

$$0,2\alpha^2 + 1,8 \times 10^{-5} \alpha - 1,8 \times 10^{-5} = 0$$

$$\alpha = 0,0038 \longrightarrow 0,38\%$$